1. Tax Incidence

This section corresponds to Nicholson pages 407 and 408. If the government imposes a per-unit tax on trade, then the price that is paid by consumers will not be the same price that is received by suppliers. When you buy a CD for $17.00, the music store only receives 91.75% of this, or $15.60, with the remaining $1.40 going to the state government as a sales tax. Therefore we need to distinguish between the two different prices. Denote the price paid by consumers as $P_D$ and the price received by suppliers as $P_S$. We use the letter $t$ for tax. Therefore,

$$P_D - P_S = t$$

Notice that if $t = 0$, $P_D = P_S$, just like usual. The goal of this section is to examine the change in $P_D$ or $P_S$ with the imposition of a tax. Therefore we are interested in finding $dP_D dt$. We use the differentials/elasticities approach as we have done before. First, notice that if the tax changes by a small amount, $dt$, then the prices will have to adjust to keep the equality above.

(1.1) \[ dP_D - dP_S = dt \]

This does not mean that $dP_D$ and $dP_S$ have to change by the same amount, just that the difference of their changes must equal the change in tax. Since we are concerned with equilibrium values we must also have,

(1.2) \[ Q_D(P) = Q_S(P) \quad \implies \quad dQ_D(P) = dQ_S(P) \]

(1.3) \[ \frac{\partial Q_D(P)}{\partial P} dP_D = \frac{\partial Q_S(P)}{\partial P} dP_S \]

We can rewrite Equation (1.1) as follows,

$$dP_S = dP_D - dt$$

We substitute this equation into Equation (1.3) to get,

(1.4) \[ \frac{\partial Q_D}{\partial P} dP_D = \frac{\partial Q_S}{\partial P} (dP_D - dt) \]
Given this we wish to solve for $\frac{dP_D}{dt}$. From Equation (1.4),

$$\frac{\partial Q_D}{\partial P} dP_D = \frac{\partial Q_S}{\partial P} dP_D - \frac{\partial Q_S}{\partial P} dt$$

$$\frac{\partial Q_S}{\partial P} dP_D = \frac{\partial Q_S}{\partial P} dt$$

$$dP_D = \frac{\partial Q_S}{\partial P} \left(\frac{\partial Q_S}{\partial P} - \frac{\partial Q_D}{\partial P}\right)$$

Now multiply the right hand side by $1 = \frac{P}{Q}$,

$$dP_D = \frac{\partial Q_S}{\partial P} \cdot \frac{P}{Q}$$

So that now we have elasticities,

$$dP_D = \frac{\frac{\partial Q_S}{\partial P}}{e_{S,P} - e_{Q,P}}$$

This means that the change in the price that the consumer pays when the tax changes is a function of the price elasticities of supply and demand. Take the extreme case for example, where customers are perfectly inelastic. This means that they do not care what price it costs, they’re going to buy it. This means that $e_{Q,P} = 0$. Referring to our equation above this means,

$$dP_D = \frac{\frac{\partial Q_S}{\partial P}}{e_{S,P} - 0} = 1$$

What does this mean? If the tax increases by 1, than the price that the consumer pays increases by one. If the tax increases by 2, then the consumers’ price increases by 2. Therefore, the consumer is bearing all of the burden of the tax. This comes from the elasticities.

2. Exercise 15.1 with an Extension

This section solves Exercise 15.1 in the Nicholson text plus an extension to the comparative static exercise we just completed above. We are given the following supply and demand functions,

$$Q_D = 1,000 - 5P$$
$$Q_S = 4P - 80$$
2.1. **Equilibrium.** To find the equilibrium quantity and price we simply refer to what an equilibrium means,

\[ Q_D = Q_S \]
\[ 1,000 - 5P = 4P - 80 \]
\[ 1080 = 9P \]
\[ P = \frac{1080}{9} \]
\[ P^* = 120 \]

Substitute this back into the formula for \( Q_D \) or \( Q_S \),

\[ Q = 1000 - 5 \cdot 120 \]
\[ Q = 1000 - 600 \]
\[ Q = 400 \]

So what is the total amount of money spent in equilibrium? Each unit costs 120 and 400 units are traded, so,

\[ \text{total spent} = 120 \cdot 400 = 48,000 \]

Now we wish to find the consumer surplus. To do this it is best to graph \( Q_D \) and \( Q_S \). Remember that we must invert the functions to draw them in our normal manner with \( Q \) on the horizontal axis and \( P \) on the vertical axis. Therefore, we have,

\[ Q_D = 1,000 - 5P \]
\[ 5P = 1,000 - Q_D \]
\[ P(Q_D) = 200 - \frac{1}{5}Q_D \]

and

\[ Q_S = 4P - 80 \]
\[ 4P = Q_S + 80 \]
\[ P = \frac{1}{4}Q_S + 20 \]

See Figure 1 for this equilibrium. To find consumer surplus we calculate the area of \( ABP^* \). In this case because the supply and demand functions are linear the area is triangular so we can use the formula for the area of a triangle,

\[ \text{area} = \frac{1}{2} \cdot \text{base} \cdot \text{height} \]

Therefore consumer surplus is given by,

\[ \text{Consumer Surplus} = \frac{1}{2} \cdot 400 \cdot (200 - 120) \]
\[ = \frac{1}{2} \cdot 400 \cdot 80 \]
\[ = 16,000 \]
Likewise,

\[
\text{Producer Surplus} = \frac{1}{2} \cdot 400 \cdot (120 - 20) = \frac{1}{2} \cdot 400 \cdot 100 = 20,000
\]

Then the total surplus is just 16,000+20,000=36,000.

2.2. **Trade Restriction.** Now assume that there is a restriction to trade (a tax for example) and now only \( Q = 300 \) units are traded. We wish to find out how much consumer and producer surplus is lost. This is the deadweight loss. In Figure 2, this is the area \( FBG \).

First, what is the surplus loss to the consumers? We need to compute the price that consumers would pay for 300 units. Referring to \( Q_D \), we have,

\[
Q_D = 300 = 1000 = 5P \\
5P = 700 \\
P = 140
\]

Therefore the top half of area \( FBG \) is

\[
\frac{1}{2} (400 - 300)(140 - 120) = 1000
\]
For producer surplus loss we must find the price that suppliers would accept for 300 units. We have,

\[ Q_S = 300 = 4P - 80 \]
\[ 4P = 380 \]
\[ P = 95 \]

Therefore the area on the bottom half of \( FBG \) is,

\[ \frac{1}{2}(400 - 300)(120 - 95) = 1250 \]

Therefore the total loss, or deadweight loss, from the restriction of trade is 1000 + 1250 = 2250.

### 3. Division of Loss

Suppose that at \( Q = 300 \), the price is 140. How does our analysis of surplus change? Now consumer surplus is \( AFH \) and producer surplus is \( HFGD \). How does this compare with what we found before? Consumer surplus can be computed as follows,

\[ \frac{1}{2} \cdot 300 \cdot (200 - 140) = 9000 \]

And producer surplus? It is the original producer surplus minus the original DWL to producers plus the rectangle \( HFGJ \). Therefore, we compute these areas,

\[ 20,000 - 1,250 + (300 \cdot 20) = 24,750 \]

Now on the other hand, assume that at \( Q = 300 \) the price is 95. How will this change the share of loss? Consumer surplus is \( AFGJ \), the original consumer surplus minus
the consumer DWL plus the same rectangle. This is,
\[ 16,000 - 1,000 + (300 \cdot (120 - 95)) = 22,500 \]
Here producer surplus is the area \( JGD \). This is computed as follows,
\[ \frac{1}{2} \cdot 300 \cdot (95 - 20) = 11,250 \]

First notice that in both cases the total surplus is the same, 33,750. However with the high price, the producers gain the majority of the surplus, whereas with the low price, the consumers gain the surplus. This should be obvious.

3.1. **Another Trade Restriction.** Now assume that the quantity is \( Q = 450 \). What is the surplus loss in this case? See Figure 3. First we need to find the prices at points \( F \) and \( G \).

![Figure 3. Another Trade Restriction](image)

\[ Q_D = 450 = 1000 - 5P \Rightarrow P_D = 110 \]
\[ Q_S = 450 = 4P - 80 \Rightarrow P_S = 132.50 \]

Notice that we are forcing trade at a higher output than under equilibrium. So at \( Q = 450 \), the suppliers wish to be paid 132.50, but the demanders only wish to pay 110. For a price of 120, the consumers are forced to pay a higher price than they would choose to, and suppliers are receiving less. Therefore the deadweight loss is the area \( BFG \). We calculate this area as usual, given the triangular formula,
\[ \frac{1}{2} (132.50 - 110)(450 - 400) = 562.50 \]

By the same arguments above, whether the price is 110 or 132.50, the total deadweight loss is the same.
4. An Extension of 15.1

This section ties the first section concerning comparative statics with example 15.1. Recall the formula we found for the change in demand price with respect to a tax.

\[
\frac{dP}{dt} = \frac{e_{S,P}}{e_{S,P} - e_{Q,P}}
\]

Let’s see how this applies to Exercise 15.1. First, we need to compute the elasticities,

\[
e_{S,P} = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = \frac{4P}{4P - 80}
\]

\[
e_{Q,P} = \frac{\partial Q_D}{\partial P} \cdot \frac{P}{Q} = \frac{-5P}{1000 - 5P}
\]

Therefore,

\[
\frac{dP}{dt} = \frac{\frac{4P}{4P - 80}}{\frac{4P}{4P - 80} - \frac{-5P}{1000 - 5P}}
\]

For our equilibrium price of 120, we have,

\[
\frac{dP}{dt} = \frac{\frac{480}{400}}{\frac{480}{400} + \frac{400}{400}} = .444
\]

This means that for every tax dollar the price demanded increase 44.44%. So if we wished to set the price demanded at \(P_D = 140\) as we had before, what tax should we set? At equilibrium the price is 120 and so we wish to raise the price by 20. Therefore, solve for \(t\) in the following,

\[\frac{444t}{100} = 20\]

\[t = \frac{20}{.444} = 45\]

Notice that part b of Exercise 15.1 effectively does this by reducing output from 400 to 300. The difference in price demanded from price supplied is, 140 - 95 = 45.

Now introduce a second demand function that is more inelastic.

\(Q_D' = 520 - P\)

This function goes through the same equilibrium point of \(Q = 400, P = 120\). What will happen to price demanded if a tax of 45 dollars is imposed? We use our formula that we derived in section 1 again. First, compute the elasticity of this new demand function,

\[
e_{Q',P} = \frac{-P}{520 - P}
\]

Substituting this into our equation, we have,

\[
\frac{dP}{dt} = \frac{\frac{4P}{4P - 80}}{\frac{4P}{4P - 80} - \frac{-P}{520 - P}}
\]

Plugging in our equilibrium value for \(P = 120\),

\[
\frac{dP}{dt} = .80
\]
Notice that this means that for every dollar increase in the tax, the demand price rises by 0.80 dollars. Compare the effect of a $45 tax on the price demanded for the two different demand curves. In the first case,

$$\frac{dP_D}{dt} = 45 \times 0.444 = 20$$

Whereas in the second case,

$$\frac{dP_{D'}}{dt} = 45 \times 0.8 = 36$$

See Figure 4. Notice that using the second demand curve we have $P_{D'} = 156$ at $Q = 364$ and the deadweight loss is the area $ABE$. Compare this to our original deadweight loss of $GBF$. Clearly $ABE$ is smaller than $GBF$. Also note that the top half of $ABE$ is the loss to the consumers, and it is the larger portion of the triangle. So total DWL is less with a less elastic demand curve, but the consumers bear a greater proportion of the loss. The smaller loss under inelastic demand curves is why taxes levied on goods for which consumers have inelastic preferences, such as cigarettes and gasoline, are less harmful to total surplus than taxes on goods such that consumers have elastic preferences.

![Figure 4. Elasticity and Taxation](image-url)